Autumn Semester 2017–2018

MATH41071/MATH61071 *Algebraic topology*

Problems 2: Simplicial Surfaces

1. Let $v_i = e_i \in \mathbb{R}^6$, the standard basis vector, for $1 \leqslant i \leqslant 6$. Let K be the set of twelve triangles:

$$K = \{\, \langle v_i, v_j, v_k \rangle \mid (i,j,k) = (1,2,4), (1,2,5), (1,3,4), (1,3,6), (1,4,5),$$

$$(1,4,6), (2,3,5), (2,3,6), (2,4,5), (2,5,6), (3,4,6), (3,5,6) \,\}.$$

Which of the conditions for a geometric simplicial surface does K satisfy?

2. Suppose that K is a simplicial surface. Use the link condition to show that each edge of K lies in precisely two triangles.

3.(*) Let $V = \{\, v_i \mid 1 \leqslant i \leqslant 9 \,\}$ where $v_i = \varepsilon_i \in \mathbb{R}^9$. For the following geometric simplicial surfaces K,

(i) verify that K is a simplicial surface,

(ii) represent $|K|$ as a polygon with edges identified in pairs and hence represent $|K|$ by a symbol,

(iii) reduce the symbol to canonical form and hence identify $|K|$.

$K = \{\, \langle v_i, v_j, v_k \rangle \mid (ijk)$ is in one of the following lists $\,\}$.

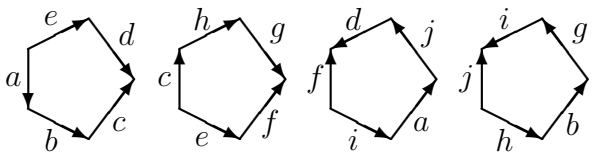
(a) $(124), (125), (134), (135), (236), (238), (246), (259), (289), (347), (358), (367), (457), (459), (469), (578), (678), (689)$;

(b) $(123), (124), (134), (234)$;

(c) $(123), (125), (136), (145), (146), (234), (246), (256), (345), (356)$;

(d) $(124), (126), (134), (137), (156), (157), (235), (237), (245), (267), (346), (356), (457), (467)$.

4. (*) The boundaries of four discs are identified as shown below.



Find a symbol for the resulting closed surface. By reducing the symbol to canonical form, or otherwise, identify the surface up to homeomorphism.

5. The interiors of three closed discs of radius 1 with centres at the points -3, 0 and 3 are removed from the closed disc of radius 5 centre the origin in $\mathbb{C}$. The boundary circles of the resulting figure are identified in pairs according to the identifications:

$$-3 + e^{i\theta} \sim 3 + e^{i\theta}, \qquad e^{i\theta} \sim 5e^{-i\theta}.$$

Find a symbol for the resulting surface. Hence, by reducing the symbol to canonical form, determine the genus and orientability type of the surface.

6. (fun) Consider the Klein Bottle template from the website. Observe, that the the Klein bottle is represented by three topological polygons with edges identified in pairs (we are ignoring the gluing tabs). Note, that one pair of edges does not get identified, although it should be (this reflects the fact, that the Klein Bottle is not realisable in 3-space without self-intersection). Find the corresponding surface symbol and reduce it to normal form. You may save time by observing that one gets three simplified polygons by applying a lot of operations of type (v) from Proposition 2.21. Hence, you may start with the simplified polygons straight away.