

In-class problems: Week 10

1. Prove that if a subspace $A \subset X$ of a topological space X is a retract of X then, for $a_0 \in A$,
 - (a) $i_*: \pi_1(A, a_0) \rightarrow \pi_1(X, a_0)$ is a monomorphism, where $i: A \rightarrow X$ is the inclusion map; and
 - (b) $r_*: \pi_1(X, a_0) \rightarrow \pi_1(A, a_0)$ is an epimorphism, where $r: X \rightarrow A$ is a retraction.
2. Prove that if $x \in D^2 \setminus S^1$ then S^1 is a retract of $D^2 \setminus \{x\}$.
[As a first step describe the retraction map informally]
3. Deduce that the set of points $x \in D^2$ such that $D^2 \setminus \{x\}$ is simply connected is S^1 .
4. Deduce that if $f: D^2 \rightarrow D^2$ is a homeomorphism then $f(S^1) = S^1$.