

In-class problems: Revision

1. Let X be the subset of \mathbb{R}^2 with the usual topology defined by

$$\{x \in \mathbb{R}^2 \mid |x - (0, 1)| = 1 \text{ or } |x - (0, 1)| = -1\}.$$

What are the path-components of $X \setminus (0, 0)$? Justify your answer.

2. Show that

$$\{x \in \mathbb{R}^2 \mid 0 < |x| < 1\} \cong (0, 1) \times S^1.$$

3. Show that $[0, 1]/\{0, 1\} \cong S^1$.

4. Consider two points $x \in X$ and $y \in Y$ in two disjoint path-connected subsets $X, Y \subset \mathbb{R}^n$. Consider their wedge-sum

$$X \vee Y = (X \cup Y)/\{x, y\}.$$

Show that $X \vee Y$ is path-connected.

5. Given an example for a topological space, which is not Hausdorff.
6. Show that the product space $X \times Y$ of two Hausdorff spaces X and Y is Hausdorff.
7. Prove that S^1 is homeomorphic to the one-point compactification of $(0, 1)$.
8. Prove that $\{-1, 1\}$ is not a retract of \mathbb{R} .
9. What is the fundamental group of $\mathbb{R}^2 \setminus \{0\}$?