

In-class problems: Week 3

1. Find all topologies on a set of three elements (say $X = \{a, b, c\}$) and divide them into homeomorphism classes.
2. Which of the following collections of subsets, together with the empty set and \mathbb{R} are a topology on \mathbb{R} ?
 - (a) the finite subsets of \mathbb{R} (*not* the finite intervals);
 - (b) the subsets of \mathbb{R} whose complements are finite;
 - (c) all subsets of the form $(a, \infty) = \{x \in \mathbb{R} \mid x > a\}$;
 - (d) all subsets of the form $[a, \infty) = \{x \in \mathbb{R} \mid x \geq a\}$;
 - (e) all subsets $U \subset \mathbb{R}$ such that $0 \in U$;
 - (f) all subsets $U \subset \mathbb{R}$ such that $0 \notin U$.
3. Show that the discrete topology on $\mathbb{Z} \subset \mathbb{R}$ coincides with the usual topology.
4. Let $X = \{a, b\}$, which topologies on X are path-connected?