

In-class problems: Week 4

1. Find subspaces X_1 and X_2 of a topological space X with $X_1 \cup X_2 = X$ and continuous functions $f_i: X_i \rightarrow Y$, to a topological space Y such that $f_1(x) = f_2(x)$ for all $x \in X_1 \cap X_2$ such that the function $f: X \rightarrow Y$ given by $f(x) = f_i(x)$ for $x \in X_i$ is not continuous.
2. Assume that X and Y are discrete spaces. Prove that the product space $X \times Y$ is discrete as well.
3. Consider the topological space (X, τ) with $X = \{a, b, c\}$ and $\tau = \{X, \emptyset, \{a, b\}, \{c\}\}$. State the product topology on $X \times X$.