In-class problems: Week 5

1. Prove that the product space $\mathbb{R} \times \{0, 1\}$, where $\mathbb{R}$ has the usual topology and $\{0, 1\}$ has the indiscrete topology, is path-connected.

2. Consider the topological space (X, τ) with $X = \{a, b, c, d\}$ and $\tau = \{X, \emptyset, \{a, b\}, \{c, d\}\}$ and the equivalence relation $\sim$ with $x \sim x$ for $x = a, b, c, d$ and $a \sim d$. State the quotient topology on $X/\sim = \{[a], [b], [c]\}$.

3. Show that $(S^1 \times [0, 1])/(S^1 \times \{0\}) \cong D^2$.