In-class problems: Week 6

1. Proof that a topological space with discrete topology is Hausdorff. What about the indiscrete topology?

2. Wich of the following topologies on $\mathbb{R}$ are Hausdorff?

(a) the subsets of $\mathbb{R}$ whose complements are finite and $\mathbb{R}$ and $\emptyset$;

(b) all subsets of the form $(a, \infty) = \{ x \in \mathbb{R} \mid x > a \}$ and $\mathbb{R}$ and $\emptyset$;

(c) all subsets $U \subset \mathbb{R}$ such that $0 \in U$ and $\emptyset$;

(d) all subsets $U \subset \mathbb{R}$ such that $0 \notin U$ and $\mathbb{R}$.

3. (a) Show that singletons are closed in a Hausdorff space.
[*Hint: write $X \setminus \{x\}$ as a union of open subsets.*]

(b) Show that a finite Hausdorff space is discrete.

[*Hint: Show that every subset is closed (this implies every subset is open).*]