# In-class problems: Week 8

**1.** Recall that a continuous function $f\colon X \to Y$ of topological spaces induces a function $f_*\colon \pi_0(X) \to \pi_0(Y)$ by $f_*([x]) = [f(x)]$. Which of the following assertions are true in general? Give a proof or counterexample for each.

(a) If $f$ is surjective then $f_*$ is surjective.

(b) If $f$ is injective then $f_*$ is injective.

(c) If $f$ is bijective then $f_*$ is bijective.

**2.** (a) Given a path $\sigma\colon I \to X$ from $x_0$ to $x_1$ in a topological space $X$, prove that

$$\sigma * \varepsilon_{x_1} \sim \sigma.$$

(b) Given two homotopic paths $\sigma_0 \sim \sigma_1$ from $x_0$ to $x_1$ in a topological space $X$, prove that $\overline{\sigma}_0 \sim \overline{\sigma}_1$.