In-class problems: Week 9

1. Given topological spaces X and Y with points $x_0 \in X$ and $y_0 \in Y$, let $p_1\colon X \times Y \to X$ and $p_2\colon X \times Y \to Y$ be the projection maps. Prove that the function

$$\pi_1\big(X \times Y, (x_0, y_0)\big) \to \pi_1(X, x_0) \times \pi_1(Y, y_0)$$

given by $\alpha \mapsto \big((p_1)_*(\alpha), (p_2)_*(\alpha)\big)$ is an isomorphism.

2. Which of the following assertions about a continuous map $f\colon X \to Y$ and the induced homomorphism $f_*\colon \pi_1(X, x_0) \to \pi_1\big(Y, f(x_0)\big)$ (where $x_0 \in X$) are true in general? Give a proof or counterexample for each.

(a) If f is surjective then f_* is surjective.

(b) If f is injective then f_* is injective.

(c) If f is bijective then f_* is bijective.