Coursework

This coursework will count 10% of the marks. There are 30 marks for the coursework from the two questions overleaf. All questions should be attempted. Marks will be given for the quality of the mathematical writing. The work should be handed in until 15:00 on Friday 17 November to the Teaching and Learning Office Reception or to my office 1.106a both in the Alan Turing Building or, alternatively, via email until 23:59 on the same day.

Family name: ....

other names: ....

Registration Number: ....

Degree programme: ....

Year of programme: ....

Course code: MATH41071 MATH61071
(encircle)

I confirm that the work submitted is my own work.

Signature: ....

Date: ....

1. (a) Let v_i be the ith standard basis vector in $\mathbb{R}^{10}$, $1 \leqslant i \leqslant 10$. Consider the set K of twenty triangles with vertices v_i , v_j and v_k where (i, j, k) is one of

$$(1,2,7),(1,2,10),(1,4,7),(1,4,10),(2,3,5),(2,3,9),(2,5,10),(2,7,9),(3,4,5),(3,4,8),$$

$$(3,6,8),(3,6,9),(4,5,7),(4,8,10),(5,6,7),(5,6,10),(6,7,8),(6,9,10),(7,8,9),(8,9,10).$$

Verify that K is a simplicial surface.

(b) Find a symbol which represents the simplicial surface K and reduce this to canonical form. Hence, identify the underlying space of K up to homeomorphism as a space in the classification theorem for closed surfaces.

(c) Confirm your answer to (b) by calculating the Euler characteristic of K and determining whether it is orientable.

[15 marks]

2. *A topological manifold with boundary* of dimension d is defined a Hausdorff, second countable topological space X, which is locally homeomorphic to $H^+ = \mathbb{R}^{d-1} \times [0, \infty)$, i.e. every $x \in X$ admits an open neighbourhood which is homeomorphic to an open subset of H^+. The *boundary* of X denoted as ∂X is the set of points, where X is not locally Euclidean (or equivalently the points in the image of the hyperplane $H = \mathbb{R}^{d-1} \times \{0\}$ under the local homeomorphisms).

Note, that with this definition a manifold X is also a manifold with boundary, where $\partial X = \emptyset$.

(a) Consider the simplicial complexes below and check whether their underlying space is a surface with boundary. In each case justify your answer.

(i) K_1 consisting of the triangles $\langle 0, e_1, e_2\rangle$ and $\langle 0, -e_1, e_2, \rangle$ and their faces, where $e_1,\ e_2 \in \mathbb{R}^2$ are the standard basis vectors;

(ii) K_2 consisting of the triangles $\langle 0, e_1, e_2\rangle$ and $\langle 0, -e_1, -e_2\rangle$ and their faces;

(iii) K' consisting of all but one of the triangles of a simplicial surfaces K and all faces of these triangles.

(b) Give a suitable definition of a *simplicial surface with boundary* by relaxing the link condition such that the corresponding underlying space is a path-connected compact surfaces with boundary. Show you definition works in this way by adapting the proof of Proposition 2.5.

[15 marks]