Autumn Semester 2017–2018

MATH41071/MATH61071 *Algebraic topology*

Problems 3: Topological Invariants of Surfaces

1. Prove the identity

$$\chi(K) = v - \frac{1}{2}f$$

for a simplicial surface K with v vertices and f triangles.

2. Fill in some details of the proof of Proposition 3.4. Prove that, given two abstract simplicial surfaces K_1 and K_2 a new simplicial surface K may be constructed by removing one triangle from each simplicial surface. $V(K)$ is given by $V(K_1) \cup V(K_2)$ with the vertices of the removed triangles identified in pairs and the triangles of K are given by the remaining triangles of K_1 and K_2.

3. Fill in the details of an inductive proof of Corollary 3.5 giving the Euler characteristic of all the surfaces in the classification theorem.

4. Complete the proof of Proposition 3.10 by showing that if all but one of the triangles in a simplicial surface can be oriented so that each pair of triangles with a common edge are coherently oriented then the simplicial surface is orientable. [Hint: Consider the link condition for each vertex of the remaining triangle.]

5. Check your answers to Problems 2, Question 3 by calculating $\chi(K)$ and (if necessary) deciding whether K is orientable.